

[CHAPTER 5]

Weak Electrolytes and Buffers

Peter A Stewart †

5.1 INTRODUCTION

Weak electrolytes affect the acid-base behavior of aqueous solutions in more complicated ways than strong ions do because they introduce additional equilibria involving H^+ or OH^- . In body fluids, weak electrolytes are usually weak acids, so we shall emphasize them in this chapter. Fortunately, the analysis that produced such illuminating results in the previous chapters works equally well here, although the mathematical complexity is much greater. As promised, most of that complexity will be relegated to the Appendix of this chapter.

One result of the additional equilibria in weak electrolyte solutions is that the analysis produces very large amounts of detailed information. Rather than the three graphs that were all we needed to display the total range of acid-base behavior in a strong electrolyte solution in the previous chapter, many dozens of such graphs would be needed here to tell the complete story. We are therefore forced to select only those aspects of weak electrolyte solution behavior that are most pertinent for understanding biologically important solutions. Nothing is lost by this, however, because the procedure we must use to arrive at this small subset is all that is needed to generate the complete story also, whenever that is desired.

The acid-base behavior of any weak electrolyte solution can be understood in detail by the procedures developed in this chapter. We therefore begin by analyzing the behavior of a solution containing a single weak acid, HA, as well as strong ions. Such a solution can be viewed as the result of simply adding HA to the strong ion solutions analyzed in the previous chapter. Its analysis will lead us into the need for computers, as well as the concept of buffering, and will provide the basis for quantitative analysis of real-life solutions that also contain CO_2 (Chapters 6 and 7).

5.2 WEAK ACID SOLUTIONS: THE EQUATIONS

In addition to strong ions, water, H^+ and OH^- , a weak acid solution contains the molecular species HA and A^- . The strong ions are represented by their electrical resultant, specified by the

value of [SID], as in Chapter 4. Water concentration is high, as always, and assumed constant; its value is included in the value of K'_w for the solution. What we know about HA and A^- is that they must satisfy the two requirements of dissociation equilibrium and conservation of mass for "A." These requirements plus water dissociation equilibrium and electrical neutrality permit us, or better, require us to write the following four simultaneously valid, independent quantitative relationships between the four unknown, dependent variables, $[H^+]$, $[OH^-]$, $[A^-]$ and $[HA]$ and the two externally controlled, independent variables, [SID] and $[A_{TOT}]$.

$$\text{Water dissociation: } [H^+] \times [OH^-] = K'_w \quad (5.2.1)$$

$$\text{Weak acid dissociation: } [H^+] \times [A^-] = K_A \times [HA] \quad (5.2.2)$$

$$\text{Weak acid conservation: } [HA] + [A^-] = [A_{TOT}] \quad (5.2.3)$$

$$\text{Electrical neutrality: } [SID] + [H^+] - [A^-] - [OH^-] = 0 \quad (5.2.4)$$

Because we have four unknowns and four equations, we can solve for the unknowns by the same systematic substituting procedure we used in Chapters 3 and 4. The result, presented in the Appendix, is a set of four cubic equations. Classically, which in this case means "before computers," the quantitative analysis of acid-base phenomena came to a paralyzed halt at this point! Getting values for $[H^+]$ from its cubic equation, given values for [SID], $[A_{TOT}]$, K_A and K'_w although possible in principle, was so long and tedious a process as to be impractical. The whole process of quantitative analysis leading up to those cubic equations was therefore generally discarded as useless also.

Fortunately, the development of computers and programmable calculators has completely changed this situation. It is now just as easy to solve such equations as it is to solve a simple quadratic. The computer does all the work, and the numerical techniques work just as well with the even more complicated fourth- and higher-order equations that result when we turn our attention to solutions like blood plasma. The general procedure is presented in the Appendix to this chapter. From a practical point of view, the Appendix may be viewed as the analog for this weak acid solution of the one-line formula for solving a quadratic equation that we used in Chapter 4 to go from Equation (4.2.3) to (4.2.5).

This new fact, that computers make practical the numerical solution of previously useless equation sets, changes profoundly our ability to understand acid-base phenomena in living systems. It permits the thorough quantitative approach that is the rationale for this book and that distinguishes this analysis from conventional treatments of the subject. Throughout the rest of this book, we shall assume that computer implementation of these general techniques is always available, so that if we can write the appropriate equations, we can get numerical answers from

them.

5.3 ACID-BASE BEHAVIOR OF WEAK ACID SOLUTIONS

The computer-implemented numerical solutions to the four equations describing a weak acid plus strong ions in water present us with a large amount of information to digest. Fortunately, one of the independent variables, namely, the total amount of weak acid present, is usually rather well regulated in living systems, for reasons not directly related to acid-base considerations. We may therefore restrict our attention, at least initially, to variations in the dependent variables resulting only from changes in the other independent variable. That behavior is presented in Figures 5.1, 5.2, and 5.3. They will repay careful study, including comparison with their counterparts in the previous chapter. The following points are especially important.

1. The curves for $[HA]$ and $[A^-]$ in Figure 5.1 are surprisingly linear and symmetrical about the midpoint of the $[SID]$ range from zero to $[SID] = [A_{TOT}]$. (The degree of dissociation, α , follows a curve just like that for $[A^-]$ because α is directly proportional to $[A^-]$ when $[A_{TOT}]$ is constant.)
2. That midpoint, at which $[SID] = [A_{TOT}]/2$ can be seen from the equations to be the point at which $[HA] = [A^-] = [SID] = [A_{TOT}]/2$, $\alpha = 0.5$ or 50%, $[H^+] = K_A$, $[OH^-] = K'_w/K_A$ and $pH = pK_A$. Detailed analysis of the equations shows that these equalities are valid only so long as K_A is within an order of magnitude or so of $\sqrt{K'_w}$, ie, in the neighborhood of 10^{-7} Eq/Liter.
3. Over the $[SID]$ range from zero to $[A_{TOT}]$, $[H^+]$ is much larger, and changes more rapidly with $[SID]$ than was the case in strong ion solutions at these same $[SID]$ values. The $[H^+]$ and $[OH^-]$ curves are no longer symmetrical about the neutral point, $[H^+] = [OH^-]$ and that point no longer occurs at $[SID] = 0$. The presence of the weak acid requires that some excess strong base cations be present when $[H^+]$ and $[OH^-]$ are equal. Conventional terminology would say that some strong base is required to “neutralize” the weak acid. That suggests that $[H^+] = [OH^-]$ should occur at $[SID] = [A_{TOT}]$ which Figure 5.1 shows is far from the truth. The situation is best understood by recognizing that the $[SID]$ value at which neutrality occurs depends on the specific forms of both the $[H^+]$ and the $[OH^-]$ curves, which means it is a property of the whole system. Any change in $[A_{TOT}]$, K_A , K'_w will change it.
4. Outside the zero to $[A_{TOT}]$ range for $[SID]$, $[H^+]$ and $[OH^-]$ behave very much as they do in strong ion solutions without weak acid. When $[SID]$ is negative, $[H^+] = -[SID]$ and $[OH^-] = -K'_w/[SID]$ as before. When $[SID]$ is positive and above $[A_{TOT}]$, $[OH^-] = [SID] - [A_{TOT}]$, $[H^+] = K'_w/([SID] - [A_{TOT}])$, and the system behaves just like a strong ion only solution with the origin of the $[SID]$ axis displaced to $[A_{TOT}]$. These properties are reflected in the $[H^+]$ and $[OH^-]$ curves at the top of Figure 5.1 as well as at the ends of Figure 5.2.

The curves of Figure 5.1 and 5.2 present the quantitative relationships between the four internal or dependent variables, $[A^-]$, $[HA]$, $[OH^-]$ and $[H^+]$ and the single independent variable,

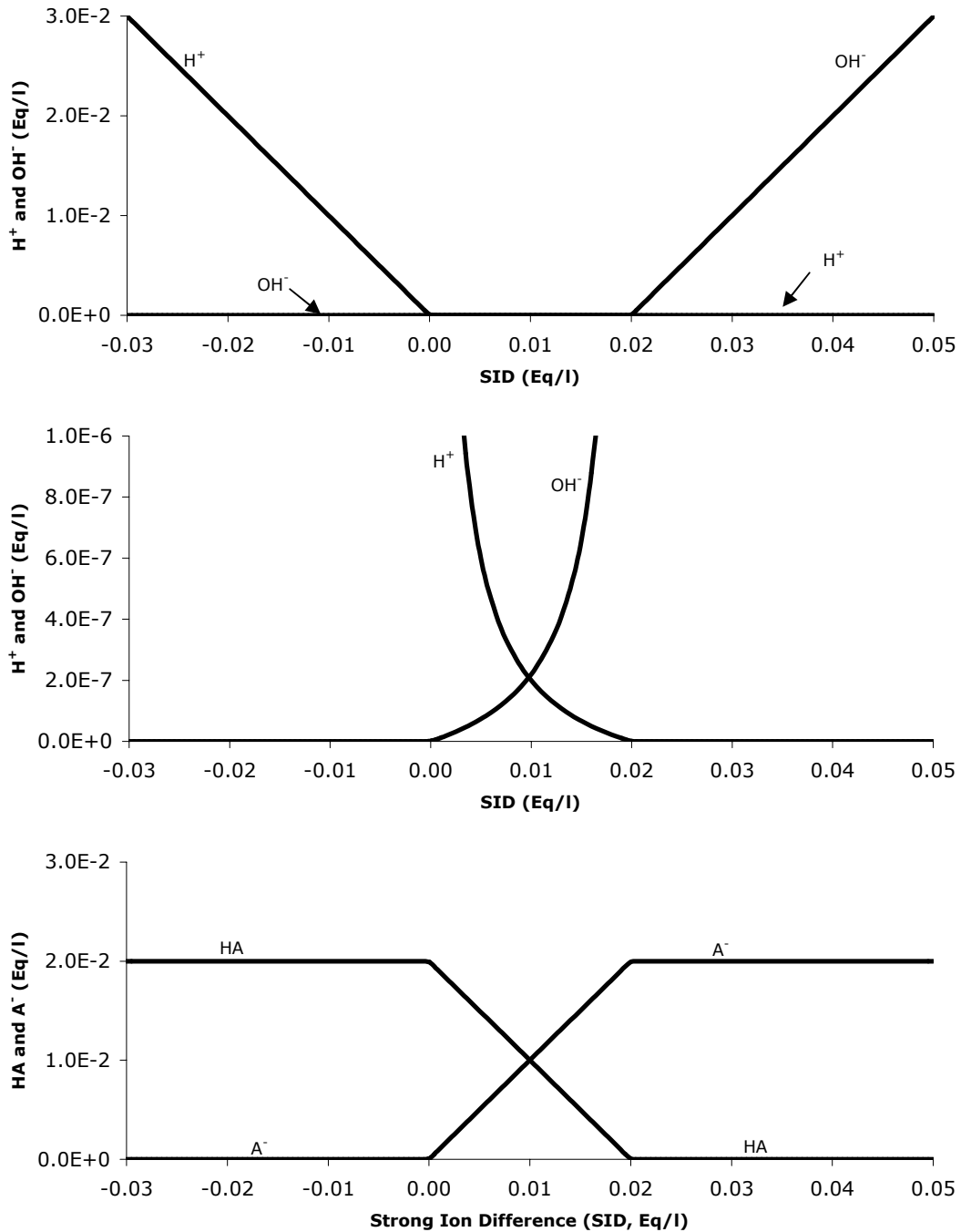


Figure 5.1. $[H^+]$, $[OH^-]$, $[HA]$ and $[A^-]$ versus $[SID]$ for a weak acid solution. $[A_{TOT}] = 0.02$ Eq/L and $K_A = 2 \times 10^{-7}$ Eq/L. The K'_w value for plasma is used, 4.4×10^{-14} (Eq/Liter)², at 37 °C. The vertical scale in the middle graph has been expanded in order to show how $[OH^-]$ and $[H^+]$ behave over the $[SID] = 0$ to $[SID] = A_{TOT}$ range. The horizontal scales for $[SID]$ are the same, and in register, for all three graphs.

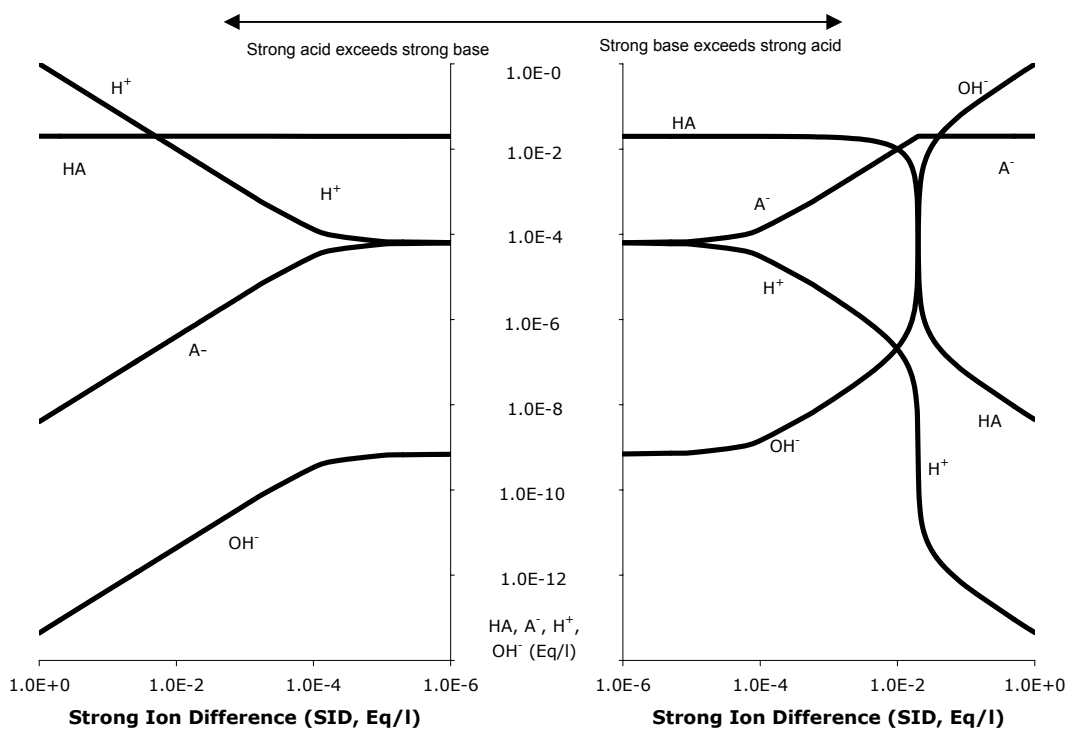


Figure 5.2. Log-log plots of $[H^+]$, $[OH^-]$, $[HA]$ and $[A^-]$ versus $[SID]$ for the same solution as in Figure 5.1.

$[SID]$, when the other independent variable, $[A_{TOT}]$ is constant, and the system is in equilibrium, satisfying Equations (5.2.1) through (5.2.4). We should also like to understand qualitatively what processes lead to these equilibrium results, and how they interact.

Suppose that in the solution described by these two figures, we begin with a $[SID]$ value of $+ 0.01$ Eq/Liter and then raise $[SID]$ by adding 0.001 Eq/Liter of KOH. The immediate result is to increase the excess strong ion positive charge by 0.001 Eq/Liter so that the net amount of weak electrolytes ions bearing negative charge must also increase by 0.001 Eq/Liter. The only weak anions available are OH^- and A^- and we have added 0.001 Eq/Liter of OH^- along with the K^+ so why does anything have to happen? Why does $[OH^-]$ not just go up by 0.001 Eq/Liter and everything else stay the same as it was? What actually happens, as the curves show, is that $[OH^-]$ hardly changes at all! It increases by less than 10^{-6} Eq/Liter, and $[H^+]$ goes down by an even smaller amount. What goes up by almost exactly 0.001 Eq/Liter is $[A^-]$.

The reason for these changes is that the added OH^- , although it could in principle serve to balance the added excess positive charge on the K^+ also has the effect of driving the water-forming reaction in the direction of more water and lowered $[H^+]$. The effect of the lowered $[H^+]$ in turn, is to drive the HA dissociation reaction in the direction of increased dissociation, thereby forming more A^- . The overall effect of the two processes interacting, therefore,

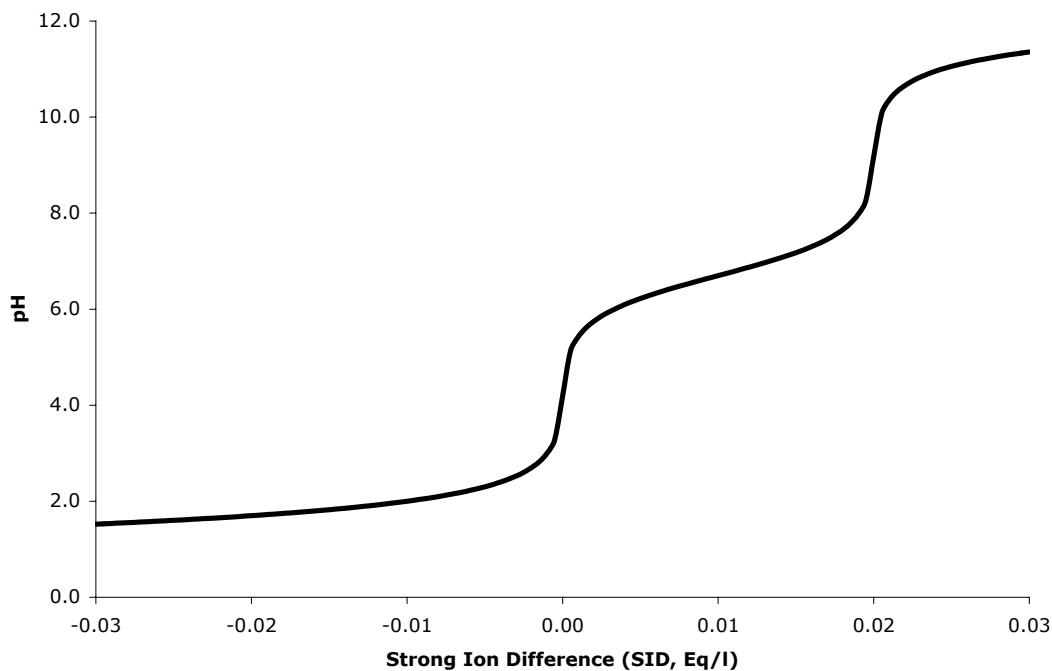


Figure 5.3. The pH versus [SID] curve for the $[H^+]$ values in Figure 5.1.

is to replace OH^- by A^- . The added OH^- is converted into water, but more HA dissociates, so that proper $[H^+]$ and charge balance are restored. Now the question becomes, why do $[H^+]$ and $[OH^-]$ change at all? Why does A^- not replace OH^- completely and leave $[H^+]$ and $[OH^-]$ just as they were? The answer lies partly in the relative magnitudes of the water and the weak acid dissociation constants and partly in the fact that this is a system, so that all four of the physical requirements on it must always be satisfied, not just electrical neutrality. A^- largely replaces OH^- because K_A is so much larger than K_w' (water is a very weak electrolyte), but it cannot do so completely because when all the reactions have settled down to equilibrium again, $[H^+]$, $[OH^-]$ and $[A^-]$ not only have to satisfy Equation (5.2.4) for electrical neutrality, they also have to satisfy the other three equations. The end result is that $[OH^-]$ is a little higher than it was, $[H^+]$ is a little lower, $[A^-]$ is almost 0.001 Eq/Liter higher, and $[HA]$ is almost 0.001 Eq/Liter lower. Within the zero to $[A_{TOT}]$ range of [SID] there is no simpler way to understand what is happening. The key point is that the water reactions are just as important as the weak acid ones. Neither one alone can explain what happens.

Outside the zero to $[A_{TOT}]$ range, there are two very different regions, just as there were before we added weak acid. When [SID] is negative, it must be balanced by positively charged weak electrolyte ions, and H^+ is the only one available, so $[H^+] = -[SID]$. When [SID] is positive and larger than $[A_{TOT}]$ it must be balanced by negatively charged weak electrolyte ions. A^- and OH^- are both available, but $[A^-]$ cannot exceed $[A_{TOT}]$ so OH^- has to fill the gap still remaining.

That is why $[\text{OH}^-] = ([\text{SID}] - [\text{A}_{\text{TOT}}])$ in this region.

As long as $[\text{SID}]$ is well within the interesting range from $[\text{SID}] = 0$ to $[\text{SID}] = [\text{A}_{\text{TOT}}]$ in a single weak acid solution such as this and K_A is within an order of magnitude of $\sqrt{K'_w}$, the following formulas may be easily derived from Equations (5.1) through (5.4) and are often useful:

$$[\text{A}^-] \cong [\text{SID}]$$

$$[\text{HA}] \cong [\text{A}_{\text{TOT}}] - [\text{SID}]$$

$$[\text{H}^+] \cong K_A \times \left\{ \frac{[\text{A}_{\text{TOT}}]}{[\text{SID}]} - 1 \right\}$$

$$\text{pH} \cong \text{p}K_A - \log_{10} \left\{ \frac{[\text{A}_{\text{TOT}}]}{[\text{SID}]} - 1 \right\}$$

$$[\text{OH}^-] = K'_w / [\text{H}^+]$$

Except for the last one, these formulas are only valid in a single weak acid solution and only useful if K_A , $[\text{A}_{\text{TOT}}]$ and $[\text{SID}]$ are known: they may be rearranged as desired if other pairs of variables than $[\text{A}_{\text{TOT}}]$ and $[\text{SID}]$ have been measured, but only if $[\text{SID}]$ is known to be within the range specified can they give usable numbers.

Most biological solutions, and all body fluids except acid gastric juice, have large positive $[\text{SID}]$ values, well above $[\text{A}_{\text{TOT}}]$ for any weak acid present, but their acid-base behavior is very different from that of Figures 5.1, 5.2, and 5.3 because they also contain carbon dioxide. We shall see in the next two chapters why that makes so much difference. From the analysis so far, the message is that whenever CO_2 is not present and the total amount of weak acid is constant, then what determines the $[\text{H}^+]$ (or pH) is the resultant of the strong ion concentrations, expressed as the $[\text{SID}]$ and how it compares with $[\text{A}_{\text{TOT}}]$.

5.4 BLOOD PLASMA WITHOUT CO_2

CO_2 is a major determinant of the $[\text{H}^+]$ in blood plasma *in vivo*, as we shall see in Chapter 7, but an isolated plasma sample without its CO_2 can still provide an interesting example of a weak acid solution such as that analyzed in the previous section. The composition of such a plasma sample is listed in Table 5.1 and may be compared with the CO_2 interstitial fluid sample described in Table 4.1 of the previous chapter. The $[\text{A}_{\text{TOT}}]$, K_A , and K'_w values in Table 5.1 have been used in the calculations that produced Figures 5.1, 5.2, and 5.3, so that Table 5.1

"HCl Added (Eq)	[Cl ⁻]	[SID]	[H ⁺]	[OH ⁻]	pH
none	0.107	0.042	2.0×10^{-12}	0.022	11.7
0.01	0.117	0.032	3.7×10^{-12}	0.012	11.4
0.012	0.129	0.020	6.6×10^{-10}	6.6×10^{-5}	9.18
0.01	0.139	0.010	2.10×10^{-7}	2.2×10^{-7}	6.70
0.01	0.149	0.000	6.3×10^{-5}	7.0×10^{-10}	4.20
0.01	0.159	-0.01	0.010	4.4×10^{-12}	2.00
0.01	0.169	-0.02	0.020	2.2×10^{-12}	1.70

Table 5.1. Titration of CO₂-free blood plasma. All concentrations in Eq/L. Initial composition: [Na⁺], 0.143; [K⁺], 0.004; [Mg²⁺], 0.002; [Ca²⁺], 0.001; [Cl⁻], 0.107; [SO₄²⁻], 0.001; [SID], 0.042; [OH⁻] 0.022; [H⁺] 2.0×10^{-12} ; [A_{TOT}], 0.020; [A⁻], 0.020; [HA], $2/0 \times 10^{-7}$; $\alpha = 100\%$. Parameter values: 37 °C; $K'_w = 4.4 \times 10^{-14}$ (Eq/L)²; $K_A = 2.0 \times 10^{-7}$ Eq/L.

may be viewed as a set of selected points from those figures. By analogy with Table 4.1, it may also be read as listing the results of titrating 1 Liter of the plasma sample with HCl. The major difference between the results in the two tables is that [H⁺] for corresponding (positive) [SID] values is larger in Table 5.1 than in Table 4.1.

The first row of Table 5.1, labeled “none,” shows that the original [SID] value in plasma is +0.042 Eq/Liter, well above [A_{TOT}]. We therefore have [OH⁻] = [SID] - [A_{TOT}] = 0.022 Eq/Liter and [H⁺] = $K'_w / 0.022 = 2.0 \times 10^{-12}$ Eq/Liter, pH 11.7. [OH⁻] / [H⁺] is above 10¹⁰, so this is an extremely alkaline solution. The weak acid is effectively completely dissociated at this [SID] value; [A⁻] = 0.02, [HA] = 2.0×10^{-7} Eq/Liter.

Adding 0.01 Eq of HCl reduces [SID] to +0.032 Eq/Liter, row 2, but this is still much larger than [A_{TOT}] so [OH⁻] falls by 0.01 Eq/Liter, [H⁺] rises by 1.7×10^{-12} Eq/Liter, while [A⁻] and [HA] do not change significantly.

Adding 0.012 Eq of HCl brings us to the [SID] = [A_{TOT}] point. The changes in [HA] and [A⁻] are still insignificant, and the change in [H⁺] is still miniscule, 6.6×10^{-10} Eq/Liter. [OH⁻] falls by slightly less than 0.012 to 6.6×10^{-5} Eq/Liter.

The next addition of 0.01 Eq of HCl puts [SID] just at the midpoint of the zero to [A_{TOT}] region, at [SID] = 0.01 Eq/Liter. Now [A⁻] = [HA] = [SID] = [A_{TOT}] / 2 = 0.01 Eq/Liter. [H⁺] = $K_A = 2.0 \times 10^{-7}$ Eq/Liter, [OH⁻] = $K'_w / K_A = 2.2 \times 10^{-7}$ Eq/Liter, and pH = pK_A = 6.66. For this system, because K_A is very close to $\sqrt{K'_w}$, this point is very close to the acid-base neutral point, at which [H⁺] = [OH⁻] = 2.1×10^{-7} Eq/Liter, pH 6.68.

One more HCl addition puts [SID] at zero. This is well on the acid side of neutrality, due to the weak acid. [H⁺] is now much larger than [OH⁻] although both are still very small. [HA] is very close to [A_{TOT}] at 0.0199 Eq/Liter, and [A⁻] is the same as [H⁺] at 6.3×10^{-5} Eq/Liter.

Further additions of HCl make [SID] more and more negative, so $[H^+]$ and $[OH^-]$ behave just as they did in Table 4.1. $[H^+]$ is just $-[SID]$, $[HA]$ is equal to $[A_{TOT}]$ almost, while $[OH^-]$ and $[A^-]$ are extremely small.

A more precise analysis of this solution would show that there are in fact many weak acids, not just one, each at a low concentration. All the K_A 's and small $[A_{TOT}]$'s could be included in a more precise set of calculations, but the net result, after an enormous amount of computer work and time, would not be significantly different from these representative results using single K_A and $[A_{TOT}]$.

5.5 BUFFERS, BUFFERING AND BUFFER STRENGTHS

“To buff” is defined in the dictionary as “to lessen the shock of,” and a “buffer” is “that which buffs.” In acid-base chemistry, a weak acid such as the HA we have been analyzing in this chapter is often referred to as a buffer. The essentially universal reliance on pH values, instead of $[H^+]$ has very much confused the quantitative aspects of buffering in weak acid solutions and has led to a misplaced emphasis on their supposed ability to “resist” changes in $[H^+]$. The quantitative analysis serves to clarify this situation very much and suggests some important restrictions on how we should think about “buffering.”

In solutions of strong ions only, considered in the previous chapter, $[H^+]$ is determined only by the value of [SID] in such a way that whenever [SID] is positive, $[H^+]$ is very small, and changes in $[H^+]$ are a minute fraction of the changes in [SID] that bring them about. (See Figures 4.1, 4.2, and 4.3.) In a solution containing a weak acid in addition to strong ions, the value of $[H^+]$ is determined by two variables, [SID] and $[A_{TOT}]$ as we have just seen. If we continue to consider only situations in which $[A_{TOT}]$ does not change, then changes in the value of $[H^+]$ can only result from changes in [SID] in these weak acid solutions also. Figures 5.1 and 5.2 show clearly that $[H^+]$ changes in response to [SID] changes are still very small, so long as [SID] is positive, but that over the [SID] range from zero to $[A_{TOT}]$, $[H^+]$ is much larger and changes much more for a given [SID] change than it does over the same [SID] range when no weak acid is present. Direct comparison of the log-log $[H^+]$ versus [SID] curves with and without weak acid in Figure 5.4 clearly demonstrates this fact. Our quantitative analysis, in other words, shows that the presence of a weak acid “buffer” actually causes $[H^+]$ to change more rapidly with changes in [SID] than when no weak acid is present.

In terms of pH, the same conclusion can be reached by comparing the pH, or titration curves of Figure 5.5. At every [SID] value between zero and $[A_{TOT}]$ the pH curve without weak acid is flatter than when weak acid is present.

Nonetheless, the presence of the weak acid does make an important difference. It changes the $[H^+]$ or pH value of the solution at any [SID] value to a more acid condition. It thus alters the $[H^+]$ or pH range that the solution experiences in response to a given range of [SID] values, so long as [SID] is positive. If, as is customary, we concentrate on the [SID] range in the neighborhood of the $[SID] = [A_{TOT}] / 2$ point, we can go further and say that the effect of the

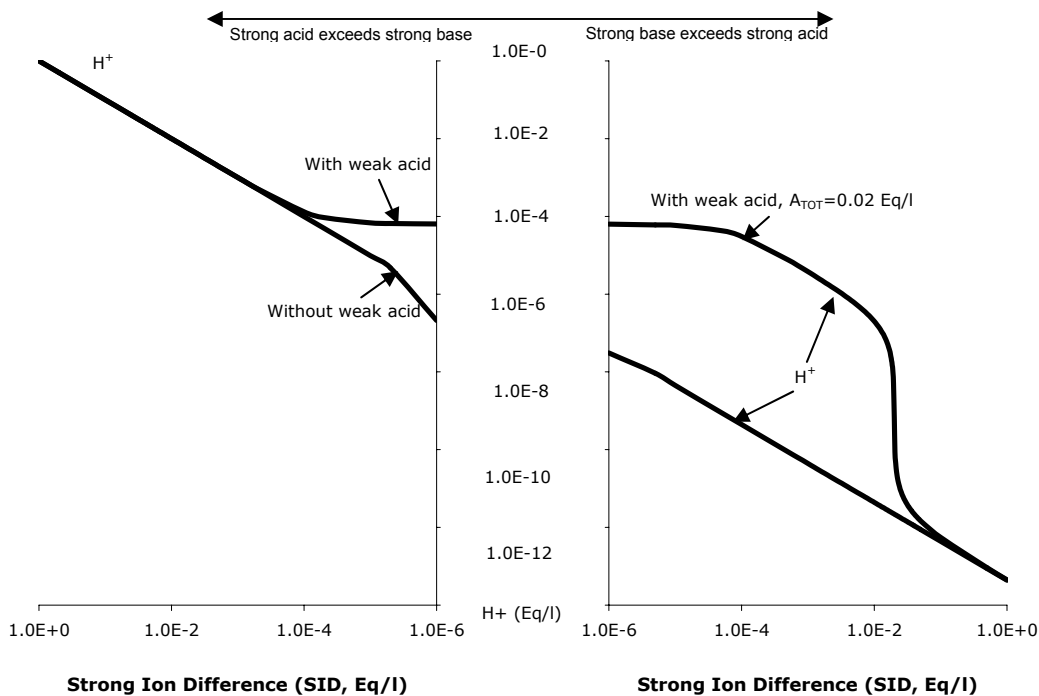


Figure 5.4. Log-log plots of $[H^+]$ versus $[SID]$ for two solutions, one with no weak acid, the other the same as in Figures 5.1 and 5.2.

weak acid is to put $[H^+]$ in the neighborhood of K_A , or pH in the neighborhood of pK_A . Without the weak acid, $[H^+]$ in this neighborhood is simply $K'_w / [SID]$, a very small value. pH is therefore large, on the order of 11 or 12. What the weak acid “buffer” does, this analysis tells us, is change the point around which $[H^+]$ changes, for a given range of $[SID]$ values. It does not decrease the amount of $[H^+]$ change; in fact it increases it. A weak acid “buffer” is therefore not an $[H^+]$ or pH “regulator” but rather an $[H^+]$ or pH “setter.” It “resists” $[H^+]$ or pH changes much less effectively than the same solution without any weak acid.

Quantitatively, the ability of a solution to “resist” changes in $[H^+]$ or pH in response to $[SID]$ changes is indicated by the slope of the $[H^+]$ or pH, versus $[SID]$ curve. The steeper the curve, the greater the $[H^+]$ or pH change in response to a $[SID]$ change, and therefore the “weaker” the buffering action. The reciprocal of this slope serves as a measure of buffer “strength,” because the flatter the curve, the larger it will be. We therefore define the following “buffer strengths”:

$[H^+]$ buffer strength = the reciprocal of the slope of the $[H^+]$ versus $[SID]$ curve for a solution.

pH buffer strength = the reciprocal of the slope of the pH versus $[SID]$ curve for a solution.

$[H^+]$ buffer strength is a dimensionless number because it represents the number of equiva-

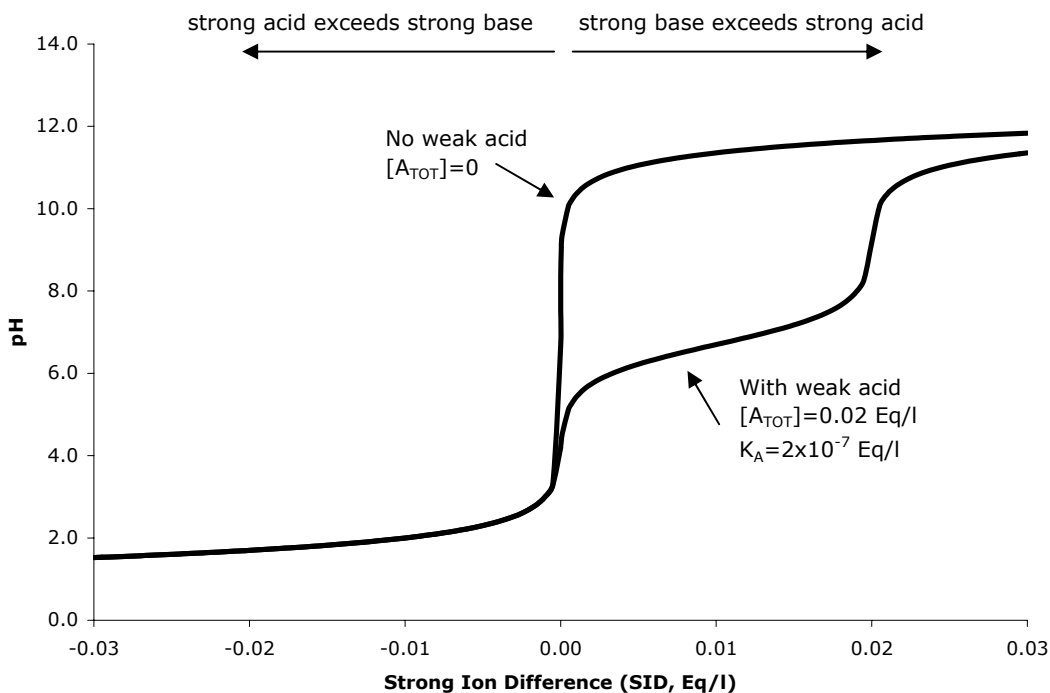


Figure 5.5. pH versus [SID] for the same two solutions as in Figure 5.4.

lents per Liter or ions per Liter, of strong ions that must be added to or removed from a solution in order to change $[H^+]$ by 1 Eq/Liter or ion/Liter. pH buffer strength, because it is the number of equivalents per Liter of [SID] change needed to bring about a pH change of 1, and pH is dimensionless, has the dimensions of [SID], equivalents per Liter.

As indicated in the Appendix, the formulas for these two quantities, given below, can be derived by straightforward application of elementary calculus to the equations for $[H^+]$ as a function of [SID] and $[A_{TOT}]$.

1. For positive [SID] values in solutions of strong ions only:

$$[H^+] \text{ buffer strength} = - \frac{[SID]^2}{K'_w} \quad (5.5.1)$$

$$\text{pH buffer strength} = 2.3 \times [SID] \text{ Eq/liter} \quad (5.5.2)$$

2. For solutions containing a weak acid, HA, at a total concentration of $[A_{TOT}]$ under the condition that [SID] is positive and very close to $[A_{TOT}]/2$: